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EXCHANGE RATE POLICIES IN A SMALL ECONOMY

The active crawling peg*

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1. Introduction

During the second half of the 1970s some Latin American countries (Argentina, Chile, Uruguay), subject to high inflation rates, adopted an exchange rate regime consisting of maintaining this rate fixed, although not constant over time, i.e. a rate independent of the behavior of other variables in the economic system. The price of foreign exchange was often preannounced in a table so that the exchange policy could be said to have adopted an 'active' attitude towards domestic inflation.

The objective of the active crawling peg is to use the exchange rate as a tool for controlling inflation in the belief that this rate is the main cause of inflationary expectations.

Such a policy therefore admits a causal relationship which is opposite to that of the 'passive' crawling peg. There the objective is to insulate foreign trade against the worst effects of domestic inflation by adjusting the rate of the crawl to purchasing power parity with the outside world. Consequently, the evolution of the passive crawl over time becomes essentially related to the behavior of other economic variables within the system.

The crawling peg policy in its passive variant was a proposal for limited exchange rate flexibility which was formally advanced during the 1960s, and independently by Murphy (1965), Williamson (1965) and Black (1966) in order to overcome the problems associated with the fixed parities system created in Bretton Woods.¹

The adjectives 'active' and 'passive' have been added by McKinnon (1979) as a way to distinguish between Williamson's proposal and the policies

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¹See Williamson (1979) for a guide to the evolution of thought about the crawling peg.

followed by different Latin American and other countries during the 1970s. The first adjective is a useful academic substitute for the popular previous notion of the 'dollar table' in southern Latin American practice during the second half of the 1970s.

This then is the empirical and conceptual background that inspired the present investigation. In what follows we *deliberately* omit any reference to existing economies since our model, as a first approach to the subject, assumes away several important short-run problems like price and wage stickiness, unemployment, and lags in the price adjustment mechanism.

Our main purpose is to formulate and answer the following question. Is it possible to prove McKinnon's conjecture (1979) regarding the active crawling peg, namely that the policy can be most useful in winding down inflation toward zero for an economy after liberalization of trade and capital movements had taken place?

The outline of the paper is as follows. Section 2 presents the model for an open two-sector economy for which traded assets are perfect substitutes. Section 3 analyzes the long-run comparative dynamics of the model under the active crawling peg policy and explores the notion of an optimal 'golden rule' rate for the crawl. Section 4 studies the local stability properties of the policy and evaluates the corresponding dynamic trajectories. The final section presents some conclusions and the appendix presents the proofs of the propositions cited in the texts. A summary of the notation employed is also provided.

2. The model

The model consists of two countries: a small home country and the rest of the world. Time is continuous.

In addition to the nominal stock of domestic money, M , residents of the home country hold two other financial assets, namely domestic and foreign bonds. These are perfect substitutes the one for the other, although both are imperfect substitutes for money.

Domestic bonds are issued by the government. Their nominal price is fixed at unity in the home currency. The net stock outstanding, D , earns interest at the current domestic nominal rate, i , and is held only by home residents.

Foreign bonds have their price fixed at unity in the foreign currency. The net stock held by home residents, F , earns interest at the current foreign nominal rate, j , fixed by conditions in the rest of the world.

Bonds are short term. No restriction is imposed on the signs of D and F ; negative quantities have an obvious interpretation as liabilities.² As a consequence of the complete integration of world capital markets, these rates are linked in the domestic market through the expected rate of the crawl, π ,

²The model ignores asset and liabilities distribution effects, following Patinkin (1965).

due to arbitrage, giving the market equilibrium condition

$$i = j + \pi. \quad (1)$$

The model assumes myopic expectations, so that the present rate of the crawl is expected to prevail. Therefore the expectation formation equation, $\pi = \dot{e}/e$, is part of the system; e represents the exchange rate. Since no jumps in the state variables are allowed, these expectations need not be rational. Total private wealth, W , is the sum of the values of these assets, $W = D + M + eF$, where foreign bonds have been converted to values in domestic currency using the current exchange rate, e .

Home residents allocate a share of their asset portfolio to money according to the portfolio behavior function, L . Market equilibrium is given by equality of the money supply with demand

$$M = L(i, A/W)W. \quad (2)$$

The first argument of L is the difference of the own real return on money — the expected rate of deflation — and the real return on its substitutes — the nominal interest rate plus the expected rate of deflation.

The second argument is the ratio of adjusted disposable income to wealth. We assume L as well as all other functions to be at least twice continuously differentiable. Its elasticities λ and ε are positive, defined in Marshall's way: $\lambda = -iL_i/L$ and $\varepsilon = (A/W)L_{(A/W)}/L$.

Nominal adjusted disposable income is used to satisfy private expenditure on goods and services, Z , and to accumulate wealth.³ The amount allocated to the latter is a positive proportion σ of the discrepancy between desired and actual wealth. Desired wealth in turn is a multiple μ of adjusted disposable income as a proxy for the idea of permanent income, so that savings are used for a gradual adjustment of wealth to its desired level at the velocity σ . This gives

$$A = Z + \sigma(\mu - AW)$$

or, solving for Z :

$$Z = A + \sigma(W - \mu A). \quad (3)$$

There are two distinctive productive sectors. One produces home or nontraded commodities for sale in the domestic markets. The other produces traded or international goods. Its output is a commodity bundle consisting of

³This is a consequence of the assumed separability of preferences over time as shown for example by Anderson and Takayama (1977).

exportables and importables. Since the relative international prices are fixed, only their net aggregate need be considered.

Productive conditions are very general, extending the well-known Salter-Swan model [Salter (1959), and Swan (1960)] expanded by Dornbusch (1972) along the lines adopted in a previous investigation [Mantel and Martirena-Mantel (1973)]. The only requirement is that the aggregate production possibility set be convex with its frontier defined by a strictly concave transformation function, and that the relative domestic prices coincide with the marginal rates of substitution in production at equilibrium. This includes as a special case the neoclassical story of fixed factor supplies, possibly some of them specific, to a given sector such as land and fixed capital, others freely transferable such as labor.

The domestic price of nontraded goods is P , and by a convenient choice of units the domestic price of traded goods is e , the exchange rate.

Denoting by X_T and X_H the quantities of the two commodities produced, the value of total output Y is

$$Y = eX_T(e, P) + PX_H(e, P). \quad (4)$$

According to the assumption on marginal pricing, the outputs are homogeneous of degree zero in their arguments. Furthermore, as is well-known, in (4) the partial derivative of the value of output with respect to one of the prices is the corresponding quantity produced.

Individual preferences for expenditure on the two goods can be described by preferences which satisfy the usual conditions. They can be described by a strictly concave utility function which for simplicity is assumed to be homogeneous, as in Dornbusch (1972). Denoting the corresponding demand functions by C_H and C_T , and given the fixed demands G_H and G_T of the government, one can describe the market equilibrium condition for home goods:

$$C_H(e, P, Z) + G_H = X_H(e, P). \quad (5)$$

Similarly, the excess demand for traded goods, net imports E_T , is:

$$E_T = C_T(e, P, Z) + G_T - X_T(e, P). \quad (5a)$$

The demand functions will of course be homogeneous of degree zero in their arguments and, due to the homotheticity of preferences, homogeneous of degree one in Z .

Multiplying (5) and (5a) by P and e and adding, we obtain the flow budget constraint of the private sector:

$$EE_T = EG_T + PG_H + Z - Y, \quad (5b)$$

where use has been made of Walras' law, i.e. $Z = EC_T + PC_H$.

Eqs. (1)–(5) describe the temporary equilibrium of the economy if we realize that the active crawling peg policy consists in fixing the time path of the exchange rate, e . Given the expected rate of depreciation and the level of the exchange rate e , the values of the predetermined variables, including exogenous data and policy variables, we can proceed to the simultaneous determination of the endogenous variables i , M , A , Z and Y .

The predetermined endogenous variables for the instantaneous equilibrium are W and P , whose paths over time will be given by the following dynamic equations.

The budget constraint for the government in the open economy is

$$\dot{M} + \dot{D} = eG_T + PG_H + iD - \theta Y + eR. \quad (6a)$$

This identity says that new issues of money, \dot{M} , and nonmonetary debt, \dot{D} , by the government and its tax receipts, θY , are used to pay for current public expenditure in the two commodities, on interest payments on the outstanding bonds, iD , and on the accumulation of foreign exchange reserves eR . The tax rate on factor income is assumed to be constant and positive; no taxes are levied on investment income. R represents the stock of foreign exchange, in foreign currency.

The net capital outflow, F , is indirectly determined due to the portfolio distribution approach, by the liquidity preference function, and by government open market operations in public bonds. If negative, it should be interpreted as a net capital inflow. The foreign exchange or balance of payments constraint in terms of foreign currency is

$$F + R = -E_T + jF, \quad (6b)$$

which says that the change in reserves equals the sum of the current account, trade surplus plus interest income from abroad, plus the capital account.

Multiplying (6b) by e and adding it to (6a) one obtains:

$$\dot{W} = (1 - \theta)Y + i(W - M) - Z \quad (6)$$

after substituting for the definition of W and replacing the trade balance by its equivalent obtained from (5b). It says that nominal wealth accumulation is the result of subtracting Z from total disposable income (unadjusted).

The second dynamic equation stems from the link between adjusted and unadjusted disposable income or, equivalently, from the estimation of the capital losses or gains relating real to nominal wealth accumulation.

Let ϕ be the consumer price index, a positive linear homogeneous function of the two prices, E and P . Real wealth will be defined as W/ϕ . Differentiating with respect to time one obtains:

$$\phi \frac{d}{dt} \frac{W}{\phi} = \dot{W} - W[\alpha \dot{P}/P + (1-\alpha)\pi], \quad (7a)$$

where α is the elasticity of the consumer price index with respect to the home goods price. Since consumption preferences are homothetic it is natural to take as consumer price index the reciprocal of the indirect utility function for unitary income, so that α is also the share of expenditure on home goods to total expenditure. This expression provides the expected change in real wealth if, as in the case of e , the economic agents also form their price expectations by equating the expected rate of inflation to the actual rate.

In stating eq. (3) real savings were assumed to be $\sigma(\mu A - W)/\phi$. Setting this expression equal to real wealth accumulation, $d(W/\phi)/dt$ in (7a), we obtain after some manipulations:

$$\alpha W \dot{P}/P = \dot{W} - \sigma(\mu A - W) - W(1-\alpha)\pi. \quad (7)$$

This relation says that short-run equilibrium in the system is obtained when the rate of inflation adjusts so as to close the gap between actual and desired real wealth accumulation. As a check, note that (7a) and (3) imply that

$$A = W + Z - W\phi/\phi.$$

On the other hand, (6) indicates that $W + Z$ is unadjusted disposable income, so that we can see that the difference between the latter and the net or adjusted disposable income is the capital loss on real wealth due to inflation, following Foley and Sidrauski (1971) and Shell, Sidrauski and Stiglitz (1969).

In other words, regarding adjusted disposable income A we can distinguish on the one hand *the formation of A* implied by the above equation

$$A = Y^d - \phi/\phi W,$$

where Y^d denotes disposable income, and on the other hand *the distribution*

of A , implied by eq. (3):

$$A = (W^d - W) + Z,$$

where W^d denotes desired wealth.

Eqs. (6) and (7) together with the five static equations suffice to describe the trajectories of the endogenous variables i , M , A , Z , Y , W and P and will therefore be referred to as the principal subsystem. The two variables, R and F , can then be obtained from (6a) and (6b) once the government has fixed its bond issues, D .

Since there is no feedback from these equations to the principal subsystem, we will focus our attention on the latter.

3. Long-run analysis

3.1. Long-run equilibrium

In order to analyze a policy which is consistent with a stationary long-run equilibrium, it will be assumed that the government adopts an active crawling peg by setting the foreign exchange table so that the exchange rate increases at a given constant rate, π . It is then simpler to replace all variables expressed in domestic currency by their corresponding values in terms of foreign currency. Notationally, capital letters — values in domestic currency — should be replaced by lower case letters — values in foreign currency — after dividing by the exchange rate e . Eqs. (1)-(7) will then be transformed as follows:

$$i = j + \pi, \quad (1')$$

this is the same equation as (1);

$$m = L(i, a/w)w, \quad (2')$$

this is eq. (2) after dividing M , A , and W by e ;

$$z = a + \sigma(w - \mu a), \quad (3')$$

obtained from (3) after dividing Z , A , and W by e ;

$$v = (1 - \theta)y(p). \quad (4')$$

Instead of presenting the value of national output as the sum of the values of the outputs of the two sectors as in (4), here we just write the after-tax return

to the factors of production in foreign currency, v , as proportional to the value of output before tax, y , which depends on the relative price of nontraded goods, p . Note that it is not necessary to specify the relation of the outputs of the two sectors to this price p , since if the productive sectors are operated efficiently we should have the profit-maximizing marginal identity

$$p\delta X_T/\delta p + \delta X_H/\delta p = 0,$$

for all relative prices, p . Therefore dividing (4) by the exchange rate, one obtains the definition:

$$y(p) \equiv y/e = X_T(1, p) + pX_H(1, p)$$

which, when differentiated, provides:

$$y'(p) = \delta X_T/\delta p + X_H + p\delta X_H/\delta p = X_H$$

and

$$X_T = y - X_H = y - py'.$$

These relations will be used in the sequel; already in the transformation of eq. (5) the evaluation of X_H in terms of the derivative of y gives as equilibrium condition for the market for nontraded goods,

$$y'(p) = g + \alpha z/p, \quad (5')$$

where g is defined to be G_H , the public demand for nontraded goods. The coefficient, $\alpha = \alpha(p)$, is the share of consumption of nontraded goods in total expenditure, so that we have:

$$\alpha = \alpha(p) = PC_H(e, P, Z)/Z = p(y'(p) - g)/z.$$

The two dynamic equations are transformed as follows. Eq. (6) becomes

$$\dot{w} = v - vm + jw - z \quad (6')$$

after dividing by the exchange rate e , substituting for y from (4'), and recalling that, thanks to (1'),

$$\dot{w} = d(W/e)/dt = \dot{W}/e = W\dot{e}/e^2 = (\dot{W} - \pi W)/e.$$

In order to deduce eq. (7') we have to differentiate (4') with respect to time,

obtaining:

$$\begin{aligned}\dot{P}/P - \pi &= d\log P/dt - d\log e/dt = d\log(P/e)/dt \\ &= d\log p/dt = (1/\psi) d\log v/dt = \dot{v}/\psi v,\end{aligned}$$

where ψ is the elasticity of the value of output with respect to the price of nontraded goods, equivalent to the share of nontraded goods in the total value of output. Substituting (7) we have, after dividing by e :

$$\alpha w \dot{v}/\psi v = \dot{w}/e - \sigma(\mu a - w) - w\pi.$$

Substituting from (6) we then obtain:

$$\alpha w \dot{v}/\psi v = v + i(w - m) - z - \sigma(\mu a - w) - w\pi,$$

so that replacing z by its value obtained from (3') and π by its value from (1) we have finally

$$\frac{\alpha w \dot{v}}{\psi} = v - im + jw - a. \quad (7')$$

It is easily checked that for a stationary solution to the system (1')-(7') — that is, a solution for which w and v are constant over time, implying that $\dot{w} = \dot{v} = 0$ — it is necessary that $a = z$, since the right-hand sides of (6') and (7') must be equal. Consequently, both a and z must equal w/μ , as can be seen from eq. (3').

The stationary solution for a fixed, given rate of crawl is depicted in fig. 1 which integrates the real and financial sectors. The vertical axis measures quantities of the traded good, whereas horizontally, toward the right of the origin O , quantities of nontraded goods are represented. The curve through point a , labelled y , represents the boundary of the production possibility set. The dotted curve through P , labelled $(1-\theta)Y$, represents the same production possibilities after deducting the proportion θ of both outputs to pay for taxes. Finally, the curve through T is the indifference curve corresponding to a unit level of utility.

In order to solve the model for the optimal rate of crawl, start with a given value for after-tax income in foreign currency, v , marked on the vertical axis. The tangent through v to the after-tax production possibility set at point P has slope $-p$ and intersects the horizontal axis at the point labelled v/p . Extend the ray issuing from O through P to point Q on the production possibility frontier to obtain the outputs of the two goods. The segment RQ represents government consumption, and therefore is fixed in length and

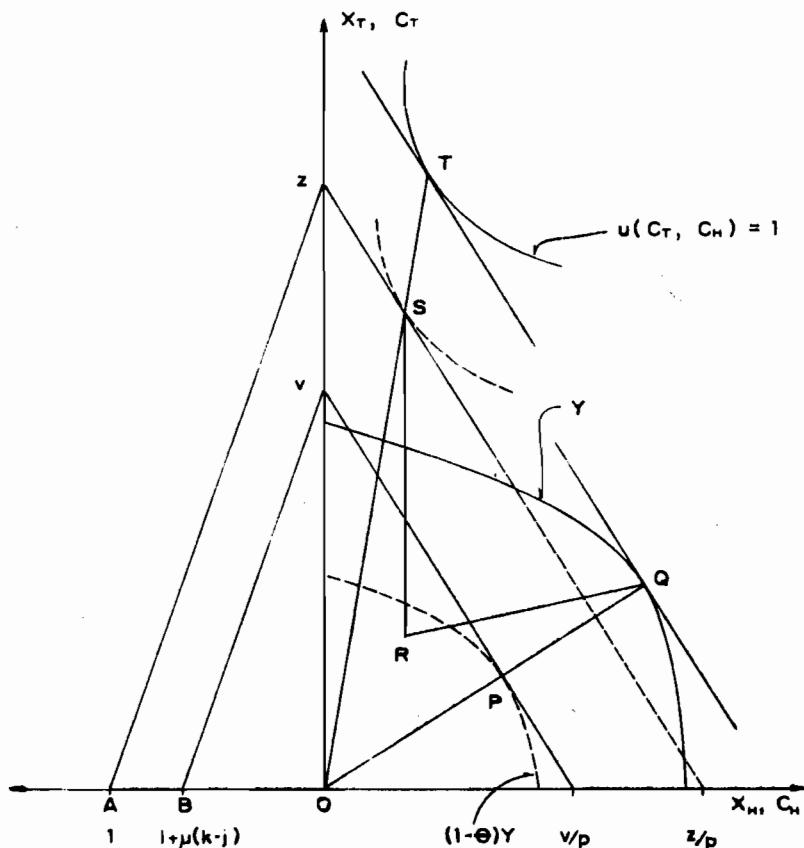


Fig. 1. Phase plane and trajectory.

slope, so that the point R represents the outputs available to the private sector.

Of course, the point representing private consumption must lie on the vertical through R , since the output of nontraded goods is consumed either by the government or by the private sector. In order to determine the said consumption point, draw the consumer expansion line \overline{OT} corresponding to prices p , i.e. the locus of points at which the indifference curves are tangent to the lines with slope $-p$. The intersection of the expansion line with the vertical through R gives the point S , indicating that \overline{RS} is the trade deficit. The tangent through S to the corresponding indifference curve intersects the vertical axis at the point labelled z .

Now draw the segment \overline{Az} , where A is some fixed point on the horizontal axis to the left of the origin, and draw the parallel to \overline{Az} through v ,

intersecting the horizontal axis at point B . By similarity of the triangles AOz and BOv , taking the length of \overline{OA} as unity, we have

$$z = \overline{Oz}/\overline{OA} = \overline{Ov}/\overline{OB} = v/\overline{OB},$$

so that from (6') we obtain for a stationary solution:

$$\overline{OB} = \frac{v}{z} = 1 + \frac{w}{z} \left(i \frac{m}{w} - j \right) = 1 + \mu(k - j). \quad (8')$$

3.2. Displacement of equilibrium and optimal rate of crawl

The effect of changes in the rate of the crawl π on the long-run equilibrium position can be analyzed in two steps. Substituting for m from eq. (2') into the dynamic relations (6') and (7'), one sees that the value of π enters the system only through the value of $k = iL(i, q)$, where by eq. (1') we have $i = j + \pi$, and $q = 1/\mu$ is the ratio of adjusted income to wealth, independent of π along a stationary path.

Consider first the effect of π on k . This can be seen to depend on the interest elasticity λ of L . Whenever the elasticity exceeds one, increases in π decrease k , whereas the opposite is true if λ is less than one, as can be inferred from the formula

$$\delta \log k / \delta \log i = 1 - \lambda.$$

Under normal conditions we would expect k to exhibit a positive minimum with respect to i . In fact, this will be so if the demand for liquidity becomes unbounded as the interest rate drops to some positive minimum rate — the liquidity trap — and at the other end there is a minimal transaction demand at all rates of interest. With these two assumptions k will have an interior minimum at which $\lambda = 1$. This of course does not exclude other local minima with $\lambda = 1$ also; the important conclusion here is that in fig. 1 point B will not be to the left of some minimal B^* corresponding to the minimizing π^* .

Secondly, once the effect of π on k has been established, consider the influence of k on relative prices and of these on welfare. The latter is the simplest relation, so we look at it first. As mentioned in section 2 real wealth is $w/\phi(p, 1)$. In the long-run equilibrium we have $w = \mu z$, with μ constant, whereas z can be determined from the nontraded good market balance eq. (5').

Thus, since α is the elasticity of the price index with respect to the price of nontraded goods,

$$\delta \log [w/\phi] / \delta \log p = \delta \log z / \delta \log p - \alpha$$

and, from (5')

$$\frac{\alpha z}{p} \left(\frac{\delta \log \alpha}{\delta \log p} + \frac{\delta \log z}{\delta \log p} - 1 \right) = p y''.$$

The elasticity of substitution in output is

$$\begin{aligned}\Sigma &= \delta \log (X_H/X_T)/\delta \log p \\ &= \delta \log (y'/(y - py'))/\delta \log p = p^2 y''/[y\psi(1-\psi)]\end{aligned}$$

and the elasticity of substitution in consumption is

$$\begin{aligned}S &= \delta \log (C_1/C_H)/\delta \log p \\ &= \delta \log ((1-\alpha)/\alpha)/\delta \log p + 1 \\ &= 1 - (\delta \log \alpha/\delta \log p)/(1-\alpha).\end{aligned}$$

Thus, one has, setting $\xi = y\psi(1-\psi)\Sigma/\alpha z$:

$$\frac{\delta \log z}{\delta \log p} = 1 + \xi - (1-S)(1-\alpha) = \alpha + \xi + (1-\alpha)S \quad (9)$$

so that

$$\frac{\delta \log (w/\phi)}{\delta \log p} = \xi + (1-\alpha)S,$$

which is positive since the substitution elasticities Σ and S are positive, and the share α is positive and less than one.

The effect of k on the relative price p can be obtained from (5') and (8). The latter equation implies

$$\mu k \delta \log k / \delta \log p = (v/z)(\psi - \delta \log z / \delta \log p).$$

Thus, p decreases as k increases if and only if

$$\psi < \alpha + \xi + (1-\alpha)S. \quad (10)$$

This relation will be assumed to be true, thus configuring what will be referred to as the normal situation in which there is some possibility of substitution in the economy. Note that it is sufficient that the sum $\xi + S$ be not less than one for (10) to hold. Relation (10) will be contradicted if

outputs and consumption patterns are independent of prices, and in addition the output of nontraded goods is lower in relation to that of traded goods than the proportions in which these commodities are desired by the consumers, i.e. if $S = \Sigma = 0$ and $X_H/X_T < C_H/C_T$.

Returning to fig. 1, this means that a displacement of point B to the left so that k increases will decrease the price of nontraded goods in foreign currency, thereby diminishing real wealth. Equivalently, the new consumption point S will lie on a lower indifference curve. It is therefore desirable to displace point B to the right as much as possible to attain its lower bound B^* corresponding to maximum real consumption expenditure, which means that k should be minimized, attaining its lower bound at $k^* = (j + \pi^*)L(j + \pi^*, q)$, with λ , the interest elasticity of liquidity preference, equal to one. By analogy with the analysis of Phelps (1967) we call the long-run real consumption expenditure maximization the golden rule for the rate of the crawl.⁴

If the assumptions about the liquidity preference function were to hold, corner solutions would be possible. In the absence of a liquidity trap, for example, it could be that k attains its minimum at $\pi = -j$ so that $k^* = 0$ and $\lambda^* \leq 1$. On the other hand if there is some interest rate i_0 which is high enough to cut off the demand for liquidity completely, then $k^* = 0$ at $\pi^* = i_0 - j$ and $\lambda \geq 1$. One of these two cases will occur in the case in which there is no interest rate at which $\lambda = 1$. The possibilities of a corner solution will be ignored in the sequel.

3.3. Local stability of long-run equilibrium

Independently of whether the active crawling peg policy follows the golden rule or not, we are interested in the stability properties of the stationary solution. The present analysis reduces to local properties. Since it is based on rather cumbersome formulae, it is confined to the appendix, where it is shown that the long-run equilibrium corresponds to a stable node if the expression $j - k(1 - \varepsilon)$ is large enough — this will surely be so if it is non-negative — and if some degree of substitution prevails in the domestic economy. However,

$$\begin{aligned} j - k(1 - \varepsilon) &= \frac{\delta}{\delta w} \left[jw - iwL\left(i, \frac{a}{w}\right) \right] \\ &= \frac{\delta}{\delta w} [i(w - m) - \pi w]. \end{aligned}$$

⁴It is interesting to notice that Mathieson (1976), with a very different model such that the law of one price exists for traded commodities as well as for financial assets and in which there are no domestic goods, obtained identical result: the optimal crawling rate corresponds to a unitary λ .

where in the last derivative m is given by the liquidity eq. (2'). This is the marginal net revenue obtained from holding additional wealth, assuming portfolio equilibrium. Its positivity indicates (individuals distribute their wealth in much the same way) that the interest earnings on nonmonetary wealth are sufficient in the margin to cover the capital loss due to inflation, which in the long run equals the rate of the crawl. The condition on the substitution elasticities is $\alpha + \xi + (1 - \alpha)S \geq 1$, which is slightly stronger than the definition of the normal case. It would be sufficient to have $S \geq 1$.

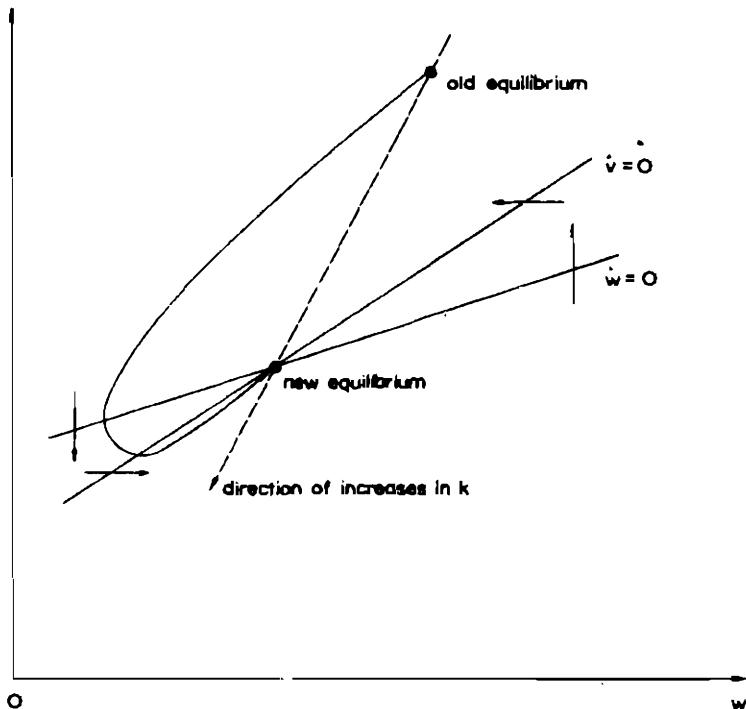


Fig. 2. Phase plane and trajectory.

In fig. 2 we give an example of a possible trajectory leading to a new long-run solution after a displacement in π . It shows the isolines $\dot{w} = \dot{v} = 0$ in the (w, v) phase plane. The long-run equilibrium at the initial π has been displaced to a new point corresponding to a higher value of k . The short-run response as drawn consists of an initial phase during which the system overshoots the target and approaches equilibrium from the opposite side. With other values for the parameters a more direct approach to the new equilibrium is possible. Nevertheless, it is shown in the appendix that the

locus of long-run equilibria has a steeper slope than the isolines, so that the initial response will always reduce both wealth and output expressed in foreign currency.

The conclusion that the system has a stable node holds under exceedingly mild conditions. It permits us to infer that a country will find itself converging very rapidly towards a lower long-run real welfare level when reducing the rate of the crawl in the face of an interest elasticity of the demand for money that exceeds unity.

4. Conclusions

Among the conclusions of the paper the following can be briefly mentioned.

(1) The active crawling peg policy in the two-sector open economy model under perfect international capital mobility and passive money was found to possess strong local dynamic convergence properties within the monetarist-Walrasian setting we have chosen in this first approach to the analysis of McKinnon's conjecture.

(2) The long-run comparative dynamics of the model allowed us to find a golden rule for the rate of the crawl or optimal crawling peg such that real wealth is maximized as a proxy for welfare. Both this result and the stability analysis allowed us to show that the system may be found to converge very rapidly towards a lower welfare level.

(3) The rate of the crawl was found to influence the composition of asset holdings in long-run equilibrium due to its effects on their relative yields. Hence, the rate depreciation of the exchange rate was found to be not neutral in the long run.

(4) The analysis has shown that in long-run equilibrium there exists unbalanced trade without the need for any change in foreign reserve holdings when interest payments are included in the analysis.

(5) The study of the trajectories followed by the endogenous variables shows the possibility of overshooting long-run equilibrium.

Much remains to be done, although to explore more closely the short-run macroeconomic properties of the policy as well as to study the active crawling peg under a setting which admits the short-run phenomena were assumed away in the introduction.

Notation

A	Adjusted disposable income in domestic currency.
$a \equiv A/e$	Adjusted disposable income in international currency.
$\alpha \equiv PC_H/Z$	Share of expenditure on home goods in total expenditure.

$C_H = C_H(e, P, Z)$	Consumption demand for nontraded goods.
$C_T = C_T(e, P, Z)$	Consumption demand for traded goods.
$\gamma = \delta \log k / \delta \log q$	Elasticity of k with respect to q .
D	Nominal stock of domestic government bonds.
E_T	Excess demand for traded goods (net imports).
e	Current exchange rate: domestic price of foreign exchange.
$\varepsilon = \delta \log L / \delta \log q$	Elasticity of liquidity preference with respect to q .
F	Foreign bonds held by domestic residents, in foreign currency.
$\phi = \phi(e, P)$	Consumer price index = reciprocal of indirect utility of unit income.
G_H	Fixed government demand for nontraded goods.
G_T	Fixed government demand for traded goods.
$g \equiv G_H$	
$\psi = \delta \log y / \delta \log p$	Share of nontraded goods in total value of output.
i	Domestic nominal interest rate.
j	Foreign nominal interest rate.
$k(i, q) \equiv iL(i, q)$	
$L = L(i, q)$	Relative liquidity preference as a function of nominal interest and q .
$\lambda = -\delta \log L / \delta \log i$	Interest elasticity of liquidity preference.
M	Nominal stock of domestic money.
$m = M/e$	Nominal stock of domestic money in foreign currency.
μ	Desired wealth as a fraction of adjusted disposable income.
P	Price of nontraded goods in domestic currency.
p	Price of nontraded goods in foreign currency.
$\pi = \dot{e}/e$	Rate of crawl.
π^*	Optimal crawl.
$q = A/W$	Adjusted disposable income as a fraction of wealth.
R	Stock of domestic reserves of foreign currency.
σ	Adjustment velocity of disposable income to savings.
$\Sigma = \delta \log(X_H/X_T)/\delta \log P$	Elasticity of substitution in production.
θ	Tax rate on factor income.
v	After-tax value of domestic output in foreign currency.
W	Total private wealth in domestic currency.
ω	Total private wealth in foreign currency.
X_H, X_T	Domestic output of nontraded and traded goods, respectively.

Y	Total value of domestic output.
$y = y(p)$	Total value of domestic output in foreign currency.
Z	Private expenditure in domestic currency.
z	Private expenditure in foreign currency.
$\eta = \mu\sigma/(1 - \mu\sigma)$	
$\xi = \Sigma y\psi(1 - \psi)\alpha z$	
$S = \delta \log(C_T/C_H)/\delta \log p$	Elasticity of substitution.

Appendix: Local stability properties

For a given rate of crawl π consider the stability of the linear approximation to the system of differential equations, (6') and (7'), in a neighborhood of the stationary solution. It will be more convenient to define new parameters ρ , η , τ , Δ in terms of which one has:

$$\begin{aligned}\mu\sigma &= \eta/(1 + \eta), \\ \mu\epsilon k &= \tau/(1 - \tau), \\ \mu(k - j) &= (\tau - \Delta)/(1 - \tau), \\ \alpha + \xi + (1 - \alpha)S - \psi &= \rho.\end{aligned}$$

The Jacobian of the system, evaluated at the stationary solution, becomes

$$\begin{pmatrix} (\Delta + \eta\tau)/\mu(1 - \tau) & -[\psi(\Delta + \eta\tau) + \rho(1 + \eta\tau)]/\psi(1 - \Delta) \\ (\Delta + \eta)\psi(1 - \Delta)/\alpha\mu^2(1 - \tau)^2 & -[\psi(\Delta + \eta) + \rho(1 + \eta)]/\alpha\mu(1 - \tau) \end{pmatrix}.$$

Necessary and sufficient conditions for the linear system with this matrix to be stable is that the trace T — i.e. the sum of the elements on the main diagonal — be negative and that the determinant D be positive. It can be shown that

$$D = \eta(1 - \Delta)\rho/\alpha\mu^2(1 - \tau)$$

and

$$\begin{aligned}T = - & [(\rho + \psi - 1)(\eta + 1) + (1 - \alpha)(\Delta + \eta\tau) + \eta(1 - \tau) \\ & + (1 - \psi)(1 - \Delta)]/\alpha\mu(1 - \tau).\end{aligned}$$

From the definitions it is seen that $\eta > 0$, $0 < \tau < 1$, $1 - \Delta = (1 - \tau)v/z > 0$, whereas in the normal case $\rho > 0$, so that the determinant D is positive. The budget and output shares satisfy $0 < \alpha < 1$ and $0 < \psi < 1$. By assumption (see section 3.3 of the text):

$$\frac{\Delta}{\mu(1 - \tau)} = j - k(1 - \varepsilon) > 0,$$

so that $\Delta > 0$, since $\mu = w/z > 0$. It is thus easily verified that sufficient for $T < 0$ is the additional condition $\rho \geq 1 - \psi$, or equivalently, substituting ρ by its definition,

$$\xi + (1 - \alpha)(S - 1) \geq 0. \quad (\text{A.1})$$

Since $\xi \geq 0$, this will be satisfied whenever $S \geq 1$; thus, if the utility function is of the Cobb-Douglas type the system will certainly be stable. Of course the condition is much weaker, permitting lower substitution elasticities in consumption provided the degree of substitution in production is adequate.

In order to investigate whether the characteristic roots of the system are real or complex, consider the ratio

$$R = -T/2\sqrt{D},$$

which is well defined and positive by the foregoing analysis. Then the roots are real if and only if $R^2 \geq 1$.

For positive real numbers a , b , and η we have the inequality:

$$(a\sqrt{\eta} + b/\sqrt{\eta})^2 \geq 4ab. \quad (\text{A.2})$$

Since T and D are linear in η , R^2 can be expressed as a function of η as in (A.2), where a and b depend on the other parameters but not on η . Thus,

$$\begin{aligned} R^2 &= [(\rho + \psi - 1)(\eta + 1) + (1 - \alpha)(\Delta + \eta\tau) + (1 - \tau) \\ &\quad + (1 - \psi)(1 - \Delta)]^2 / 4\alpha(1 - \tau)\eta(1 - \Delta)\rho \\ &\geq [(\rho + \psi - 1) + (1 - \alpha)\tau + (1 - \tau)][(\rho + \psi - 1) + (1 - \alpha)\Delta \\ &\quad + (1 - \psi)(1 - \Delta)] / \alpha(1 - \tau)(1 - \Delta)\rho. \end{aligned}$$

The last expression decreases with τ , so setting τ to its lower bound 0,

$$R^2 \geq [\rho + \psi][(\rho + \psi - 1) + (1 - \alpha)\Delta + (1 - \psi)(1 - \Delta)] / \alpha(1 - \Delta)\rho.$$

Similarly, this decreases with increasing α , so setting $\alpha = 1$:

$$R^2 \geq [\rho + \psi][(\rho + \psi - 1) + (1 - \psi)(1 - \Delta)] / (1 - \Delta)\rho.$$

This expression decreases with Δ , so we can set $\Delta = 0$:

$$R^2 \geq [\rho + \psi][\rho] / \rho = \rho + \psi - 1.$$

The last relation is of course (A.1). Thus, we can conclude that the characteristic roots are real and negative. Therefore the stationary solution is a stable node.

In order to draw fig. 2 we need the slopes of the isolines and of the equilibrium displacement line. From the Jacobian we obtain:

$$\frac{\delta v}{\delta w} \Big|_{w=0} = (\Delta + \eta\tau)\psi(1 - \Delta)/\mu(1 - \tau)[\psi(\Delta + \eta\tau) + \rho(1 + \eta\tau)] = a,$$

$$\frac{\delta v}{\delta w} \Big|_{w=0} = (\Delta + \eta)\psi(1 - \Delta)/\mu(1 - \tau)[\psi(\Delta + \eta) + \rho(1 + \eta)] = b.$$

It is easily checked that b is the expression a if τ is replaced by one. Since a is increasing in τ , we have $a < b$.

From section 3.2 of the text, especially eq. (9), we obtain for a displacement of equilibrium:

$$\frac{\delta v}{\delta w} = \frac{v}{\mu z} \frac{\delta \log v / \delta \log p}{\delta \log z / \delta \log p} = \frac{1 - \Delta}{\mu(1 - \tau)} \frac{\psi}{\rho + \psi} = c.$$

But b increases with η , and c is obtained from b by letting η tend to $+\infty$, hence $c > b$.

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